

Market-System Design Optimization With Consider-Then-Choose Models

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Design optimization in market system research commonly relies on Discrete choice analysis (DCA) to forecast sales and revenues for different product variants. Conventional DCA, which represents consumer choice as a compensatory process through maximization of a smooth utility function, has proven to be reasonably accurate at predicting choice and interfaces easily with engineering models. However, the marketing literature has documented significant improvement in modeling choice with the use of models that incorporate both noncompensatory (descriptive) and compensatory (predictive) components. This noncompensatory component can, for example, model a “consider-then-choose” process in which potential customers first narrow their decisions to a small set of products using noncompensatory screening rules and then employ a compensatory evaluation to select from within this consideration set. This article presents solutions to a design optimization challenge that arises when demand is modeled with a consider-then-choose model: the choice probabilities are no longer continuous or continuously differentiable. We examine two different classes of methods to solve optimal design problems—genetic algorithms (GAs) and nonlinear programming (NLP) relaxations based on complementarity constraints—for consider-then-choose models whose screening rules are based on conjunctive (logical “and”) rules. [DOI: 10.1115/1.4026094]

1 Introduction

Decision-based design (DBD), enterprise-system design, and value-driven design research asserts that business objectives should serve as the objective for engineering design; see, e.g., Refs. [1–3]. Wassenaar and Chen [4] have specifically proposed the use of economic value to the firm—e.g., profits—as the metric against which different designs should be judged. In order to forecast profits engineering researchers have integrated generalized linear models that predict customer choice share for potential products. Discrete choice analysis (DCA) [5–7] is most commonly used, with implementations adopting a variant of the logit or probit models [4,8–17].

The generalized linear models typically used within the DCA paradigm model consumer decisions as *compensatory*, with continuous tradeoffs between product attributes and price. Real choice behavior, however, may take a different form. Imagine a woman shopping for a new vehicle. She is considering purchasing either a sedan for commuting or a truck for taking her boat to the lake. She already owns an old sedan, so she is looking for a new one that will reduce operating and maintenance costs over a long time frame, with a reasonable price and excellent gasoline mileage. She does not own a truck, and if she decides to buy one she will be on a strict sticker-price budget to allow for continued upkeep of the old sedan. Towing capacity for the weight of the boat is required, while gasoline mileage and cost is not important as it will be used infrequently. A compensatory model would have great difficulty dealing with the disjunction of wanting either a sedan or a truck, and the additional design combinations associated with each preference. The estimation of this customer’s preferences with a typical logit model would have extremely limited predictive power. Yet, this type of purchase decision is common: considering two different classes of vehicles and having different feature preferences for each class. Given the widespread

experimental evidence of noncompensatory and/or “heuristic” decision-making strategies [18,19], it is possible that the usefulness of choice models used in engineering design hinges on inclusion of noncompensatory behavior.

Researchers in economics, psychology, and marketing have developed a variety of *noncompensatory* decision models to address such behaviors, as discussed more fully in Sec. 2. This article presents a first attempt at incorporating one such model—the hybrid noncompensatory/compensatory *consider-then-choose* model developed by Hauser and Wernerfelt [20]—into engineering design optimization. The consider-then-choose model posits that consumers use noncompensatory screening rules to limit the number of options they *consider* and then *choose* from these limited options with a compensatory evaluation [20,21]. While the marketing literature has recently expanded empirical applications in modeling consideration, it does not yet investigate optimal decision making under this model form.

This article focuses on methods for optimal design under consider-then-choose models with the simplest type of noncompensatory screening rule: conjunctive (logical “and”) rules. The associated optimal design problem is significantly more complex than with compensatory decision models because of discontinuities in the objective (profits) resulting from changes in consideration. With heterogeneous consumer models and multiple products, there may be many discontinuities. As a consequence many local optima may exist and derivative-based NLP algorithms, such as sequential quadratic programming (SQP) [22,23], interior-point (IP) methods [22,24], or augmented Lagrangian methods [22,25] may not apply or be efficient. Genetic algorithms (GAs) are a viable option as they are frequently used with discontinuous models [26]. GAs may, however, have more difficulty with precisely solving design constraints. Our purpose here is to explore the applicability of both GAs and NLP methods based on a novel relaxation. NLP methods have been successfully applied in other, related areas of large-scale mathematical programming with nondifferentiable functions [27–29] and are thus worth considering here.

While the vast majority of design literature adopting some model of consumer demand assumes a compensatory decision

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process, Besharti et al. [30] consider conjunctive and disjunctive decision-making. We add to their efforts by relying on a model form that can be statistically estimated—a topic of active research in marketing—and link this choice model directly to optimal design decision-making in a manner consistent with the majority of applications of purely compensatory DCA models.

The remainder of this article proceeds as follows: Section 2 introduces consideration behaviors, consider-then-choose models, and an associated optimal design formulation. Section 3 presents an example of vehicle design decisions, where consideration is determined by satisfaction of a budget rule. Section 4 discusses GAs and an “MPCC relaxation” that adopts techniques from mathematical programming with complementarity constraints (MPCCs). Section 5 defines an extension of the example in Sec. 3 to heterogeneous populations and multiple-product portfolios in order to compare computational methods. Both examples are illustrative in nature and include generalizations and simplifications in the models that make them unsuitable for discussion of highly detailed vehicle design results or conclusions. These examples serve to motivate further work in modeling consideration behaviors in engineering design optimization. Section 6 discusses the results, and Sec. 7 concludes the article. Mathematical notation is summarized in Nomenclature.

2 Consider-Then-Choose Models

This section provides a brief review of relevant noncompensatory choice models for the purposes of familiarizing the engineering design community with these models and giving an idea of the empirical grounding, longevity, and acceptance of this area of behavioral/choice research. We do not focus on important issues of empirical data collection and estimation of noncompensatory models and recommend readers review references provided below for more authoritative treatments than can be provided here.

2.1 Noncompensatory Choice and Consideration. One of the first concepts in non-compensatory choice was published in 1957, in which Simon proposed the principle of *satisficing* [31]: individuals search through the available options only until a suitable option is discovered, regardless of whether that option objectively maximizes utility or value. Soon after, several researchers introduced the *conjunctive model* in which an option must have a minimum value across several criteria in order to be selected [32–34]. Tversky proposed the famous *Elimination-By-Aspects* model, in which an individual focuses on a particular product attribute and eliminates all options that fail to meet some minimum criteria for that attribute, and repeats this process over all product attributes until all but one option is eliminated [35]. Sometimes, a noncompensatory model predicts a decision that is not the optimal outcome, for example in satisficing the outcome must be “good enough” not “best.” Yet this decision may be optimal with respect to balancing the amount of time and effort spent on a decision relative to its outcome [36]. This is especially true when information about product attributes is expensive to obtain or process [18,20].

Marketers expanded this early research in psychology and applied it to complex product categories in which the large number of available products makes it unlikely that a consumer carefully evaluates each and every product option. Experimental evidence for the construction and use of *consideration sets* was introduced by Payne [37]. This seminal experiment suggested that consumers first limit their choice to some viable options using screening rules and only then make a more calculated decision amongst the remaining options. This work has been followed with a long tradition in marketing; see, e.g., Ref. [18] and references therein. Including consideration is also known to significantly improve quantitative choice model accuracy [21], explaining “approximately 80% of the uncertainty in deodorant choice” [38] in one marketing study.

Noncompensatory models can now be estimated using similar techniques to traditional survey-based DCA models as detailed in

Refs. [39–45]; see also Sec. 2.2. Subjects are typically shown multiple choice sets that present a variety of product configurations. A model is then built to predict the choices observed in the experiment, and the model includes implicit determinations of consideration as well as evaluation of the relative merits of each product in the choice set. It can be difficult to derive complex screening rules from DCA-type questions, especially rules such as disjunctions and disjunctions of conjunctions; e.g., wanting a fuel-efficient, cheap-to-maintain sedan with a gas engine or a truck with a low purchase price and large towing capacity regardless of fuel efficiency. Ding et al. [46] have demonstrated that with careful incentive alignment, effective noncompensatory models can be built directly from respondent language. In these experiments, subjects are asked to write a letter to a representative that will choose a product based on the subject’s instructions. The subject is entered into a lottery to win the product he or she described, or a near approximation, providing strong incentive to describe what they want to buy accurately. It is found that these letters are particularly effective at predicting choice when written after the respondent has answered DCA-type choice set questions.

Marketing researchers have argued the benefit of noncompensatory models solely on the basis of improving predictive power over existing models, such as compensatory. However, this improvement can be marginal: some researchers have obtained only single-digit percentage points improvements (e.g., Refs. [39–41,46,47]) although others observe double-digit improvements (e.g., Refs. [43,44,48]). This argument is limited, as demonstrated here, as the real advantages of noncompensatory modeling are revealed through application to product design, namely engineering design optimization. Models with equal predictive power can lead to dramatically different product designs, associated with different profitability potential.

2.2 Consider-Then-Choose Models. In the first stage of the consider-then-choose model, the customer decides what subset of products he or she is willing to consider for purchase—the consideration set. In the second stage, the customer chooses one product from amongst this set. If a particular product is not in the set, it has a zero probability of being chosen. The products in the set are modeled as evaluated using random utility theory. We use the mathematical model described below to capture this behavior.

There are I individuals, which can either represent an individual customer’s screening rules or a group of customers’ homogeneous screening rules. Each individual $i \in \{1, \dots, I\}$ considers only the products indexed by an individual-specific consideration set $C_i(\mathbf{X}, \mathbf{p}) \subset \{1, \dots, J\}$. All available products are described by $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_J) \in \mathcal{X}^J$, and all product prices are described by $\mathbf{p} = (p_1, \dots, p_J)$. A collection of *screening rules* s_i , define the consideration set, a subset of available products, as follows:

$$C_i(\mathbf{X}, \mathbf{p}) = \{j \in \{1, \dots, J\} : s_i(\mathbf{x}_j, p_j) \leq \mathbf{0}\} \quad (1)$$

This formulation of $C_i(\mathbf{X}, \mathbf{p})$ in terms of screening rules mirrors the negative null form of inequality constraints in optimization theory. For example, “thresholding” rules require that a certain characteristic, say $x_{j,k}$, achieves at least given level γ ($x_{j,k} \geq \gamma$) for product j to be considered [39]; this can be written by taking $s(\mathbf{x}, p) = \gamma - x_k$. Marketing researchers have used indicator functions to define consideration sets [39,44,45]. The negative null form shown in Eq. (1) is equivalent to an indicator representation, and potentially advantageous for optimization.

Individuals choose which product to purchase from their consideration set $C_i(\mathbf{X}, \mathbf{p})$ following the traditional random utility maximization (RUM) paradigm used in DCA [7]. Each individual has a “systematic” utility function $u_i(\mathbf{x}, p)$ and “outside good” or “no purchase” utility ϑ_i . To include the influence of decision factors not captured in u_i and ϑ_i , individual i is assumed to choose the alternative that maximizes *random* utility $U_{i,j}$ for product $j \in C_i(\mathbf{X}, \mathbf{p})$ or Θ_i for the outside good, where

$$\begin{aligned} U_{i,j} &= u_i(\mathbf{x}_j, p_j) + \mathcal{E}_{i,j}, \quad j \in C_i(\mathbf{X}, \mathbf{p}) \\ \Theta_i &= \vartheta_i + \mathcal{E}_{i,0} \end{aligned} \quad (2)$$

for some random errors $\mathcal{E}_{i,0}$ and $\mathcal{E}_{i,j}, j \in C_i(\mathbf{X}, \mathbf{p})$. Only products in the consideration set are included in the utility maximization.

As with traditional DCA, different choices for the distribution of the errors give different models of choice probability. For example, assuming these variables to carry a joint normal distribution yields the Probit model for the second compensatory stage, which does not have closed-form choice probabilities [7]. Gilbride and Allenby describe hierarchical Bayesian procedures for simultaneously estimating parameters defining thresholds as well as parameters in the utility function for multinomial probit DCA models. We assume the errors are i.i.d. extreme value, obtaining the multinomial logit model for the compensatory stage [7]. Liu and Arora [45] discuss optimal experimental designs for Maximum Likelihood estimators for consider-then-choose models with this error specification. Dzyabura and Hauser discuss Bayesian methods for adaptive question surveys [44]. Assuming i.i.d. extreme value errors implies that the probability that individual i chooses any product $j \in \{1, \dots, J\}$, denoted $P_{i,j}^C$, can be written as

$$P_{i,j}^C(\mathbf{X}, \mathbf{p}) = \begin{cases} \frac{e^{u_i(\mathbf{x}_j, p_j)}}{e^{\vartheta_i} + \sum_{k \in C_i(\mathbf{X}, \mathbf{p})} e^{u_i(\mathbf{x}_k, p_k)}} & j \in C_i(\mathbf{X}, \mathbf{p}) \\ 0 & j \notin C_i(\mathbf{X}, \mathbf{p}) \end{cases} \quad (3)$$

The first term is the standard logit probability, which applies to products in the consideration set. Note that in the denominator we include only those products in the consideration set (and the outside good). The second term states that if a product is not in the consideration set, it has zero purchase probability. Thus, if product j satisfies individual i 's screening rules, it is chosen with a probability determined by a standard logit model defined over only those options considered; if product j does not satisfy individual i 's screening rules, it has zero probability of being chosen.

The key obstacle for product portfolio optimization is that the choice probabilities $P_{i,j}^C$ defined in Eq. (3) can be discontinuous functions of product characteristics and prices. We illustrate this mathematical fact with a simple example.

Example 1. Suppose an individual can purchase a product with price p (\$) and "value" $x \geq 0$. If they buy the product they obtain a utility of $U(x, p) = u(x, p) + \mathcal{E}$ where $u(x, p) = 1 - p + x$, and if they opt not to buy the product they obtain a utility of $\Theta = 1 + \mathcal{E}_0$ for i.i.d. extreme value errors $\mathcal{E}, \mathcal{E}_0$. However, they will consider the product only if $p \leq \$1 - a$ screening rule. This example models a situation where individuals only have so much to spend on a product and can obtain utility from both ownership and the amount of money left over after purchase. Consider-then-choose choice probabilities are then given by

$$P^C(x, p) = \begin{cases} e^{1-p+x}/(e + e^{1-p+x}) & \text{if } p \leq \$1 \\ 0 & \text{if } p > \$1 \end{cases} \quad (4)$$

(We exclude subscripts i and j because we focus on a single individual and a single product.) Observe that $\lim_{p \uparrow 1} P^C(x, p) = e^x/(e + e^x) > 0$ while $\lim_{p \downarrow 1} P^C(x, p) = 0$, and thus $P^C(x, \cdot)$ is discontinuous at $p = 1$; see also Fig. 3 in Sec. 4. \square

The discontinuity presented in the example, extrapolated to 1,000,000 individuals all with different screening rules, demonstrates an optimization challenge and a potential opportunity for market advantage. If a firm can identify and exploit screening rules and thus discontinuities in choice, they may uncover suitable designs missed by competitors relying on continuous DCA models. However optimizing designs with these discontinuities in choice may not be straightforward, a central issue motivating this article.

2.3 Difference Between Choice and Consideration Sets.

Choice sets, a common feature of DCA studies, are different from consideration sets. Consideration sets are sometimes referred to as choice sets in the literature, leading to potential confusion between the two terms. For the purposes of this article, we define *choice sets* as the sets of alternatives in multiple choice questions created and presented to subjects as during a product choice experiment. The responses to these questions are used to estimate the parameters of a decision models [6]. *Consideration sets*, on the other hand, are subsets of alternatives constructed by the consumers themselves when making a decision. The rules from which consideration sets can be constructed can be estimated from observed responses to choice set questions at the individual level as a function of product attributes, prices, and other elements of the choice context.

2.4 Optimal Design Formulation. A general formulation of optimal design with a consider-then-choose model can be written as in Eq. (5):

$$\begin{aligned} \max \quad & \pi^C(\mathbf{X}, \mathbf{p}) \\ \text{w.r.t.} \quad & \mathbf{l}_j \leq \mathbf{x}_j \leq \mathbf{u}_j, \quad p_j \geq 0 \text{ for all } j \\ \text{s.to} \quad & \mathbf{c}_j^E(\mathbf{x}_j) = \mathbf{0}, \quad \mathbf{c}_j^I(\mathbf{x}_j) \geq \mathbf{0} \text{ for all } j \end{aligned} \quad (5)$$

Terms are as follows: π^C , defined by

$$\pi^C(\mathbf{X}, \mathbf{p}) = \sum_{j=1}^J \sum_{i=1}^I P_{i,j}^C(\mathbf{X}, \mathbf{p})(p_j - c_j(\mathbf{x}_j)) - \frac{c^F}{I} \quad (6)$$

are expected profits normalized by the number of individuals I ; \mathbf{x}_j denotes the vector of design variables for product j ; $\mathbf{l}_j, \mathbf{u}_j$ are lower and upper bounds on \mathbf{x}_j ; p_j is the price of product j ; $c_j(\mathbf{x}_j) \geq 0$ is a unit cost as a function of design decisions; c^F are (constant) fixed costs for the firm; \mathbf{c}_j^E are equality constraints; and \mathbf{c}_j^I are inequality constraints. The primary difficulty in solving Eq. (5) is that π^C is discontinuous when $P_{i,j}^C$ is defined by Eq. (3). GAs have no difficulty with this mathematical feature, but may have more difficulty finding solutions that precisely satisfy the design constraints. Existing gradient-based NLP methods can precisely satisfy constraints but cannot be directly applied to a problem with a discontinuous objective. Application of both methods is developed in Sec. 4, after an illustrative example of this type of design problem in Sec. 3.

3 Example: Vehicle Purchase With an Owning and Operating Cost Budget

This section examines the design of a single personal vehicle for a homogenous population under three different conditions: (1) a typical compensatory preference model, such as might be generated from a DCA study, (2) a noncompensatory representation of the same preference, and (3) a compensatory approximation to the noncompensatory preferences. Comparing the three conditions illustrates the practical importance of including noncompensatory consideration in design as well as computational benefits of representing noncompensatory choice discontinuously. Note that the example pursued throughout the remaining sections of the paper is one of the simplest representations of a consider-then-choose model and uses a "rational" screening rule rather than the more "heuristic" screening rules typically considered in Marketing and Psychology [19].

3.1 Vehicle Design Model. In this illustrative example, the design variables are 0-60 acceleration time (a , ranging from 2.5-15s) and Manufacturer's Suggested Retail Price (p , in

\$10,000). This range of acceleration times, though extreme, reflects the range exhibited in 2006 vehicle data. Neither bound is active in any of the optimal design results. Fuel consumption (g , in gpm) is related to 0-60 acceleration time by a function $g = G(a)$:

$$G(a) = 0.035 + \frac{53.5 + 69.5e^{-a} - 1.8a^{1.4} + 106.9/a}{1000} \quad (7)$$

This equality constraint illustrates the general trend of inversely related acceleration performance and fuel economy, a trend that can be seen in recent vehicle data. This trend is also a feature of other, more complex equality constraints based on empirical models [13,49,50]. Unit costs are also a function of acceleration, given by $c(a)$:

$$c(a) = e^{a/12} \left(1.5 + 1.97e^{-a} - 0.04a + \frac{1}{a-1.5} \right) \quad (8)$$

Fixed costs of production are not modeled.

3.2 Consumer Choice Models. In order to demonstrate the usefulness of capturing and representing noncompensatory choice in design, we examine three choice model scenarios related to compensatory vs. noncompensatory choice models. Scenario (1) uses the basic logit consumer choice model

$$P(a, g, p) = \frac{e^{u(a, g, p)}}{1 + e^{u(a, g, p)}} \quad (9)$$

where the utility function u depends on acceleration, fuel consumption, and vehicle price as follows:

$$u(a, g, p) = -3.6p - 36.8g + \frac{11.3}{a} + 23.2 \quad (10)$$

Scenario (2) adds the noncompensatory screening rule associated with budgeting for annual owning and operating cost: if annual owning and operating costs are greater than a budget level B , the consumer will not consider the vehicle. This is added as a realistic example of a screening rule that can affect design decisions. The existence of popular metrics like Edmunds True Cost to Own [51], availability of monthly loan or lease payment calculators (e.g., Ref [52]), and inclusion of information about estimated annual fuel costs on mandatory EPA vehicle labels [53] suggest that owning and operating cost is an important feature to vehicle buyers. The addition of this rule also exemplifies a useful property of noncompensatory models: the incorporation of decision rules from different data sources, at least for a homogenous population. For example, a company may have a well-established budget limit rule that they do not find necessary to test in consumer decision surveys.

Annual owning and operating costs for a vehicle purchased on credit are modeled by

$$k(g, p) = Rp + mp^G g, \quad R = \frac{r(1+r)^N}{(1+r)^N - 1} \quad (11)$$

where p (in \$) is the vehicle purchase price, $r = 0.06$ is an annual interest rate, $N = 10$ is the number of loan periods, $m = 15,000$ is the number of miles driven per year, and $p^G = \$3.5$ is the price of gasoline. Ten year loans are uncommon in the automotive industry, with 3–6 year loans being more common. A shorter term would decrease individuals' purchasing power. Referring back to Eq. (1), the screening rule is $s(a, g, p) = k(a, g, p) - B$ and consider-then-choose choice probabilities are

$$P^C(a, g, p) = \begin{cases} P(a, g, p) & \text{if } k(g, p) \leq B \\ 0 & \text{if } k(g, p) > B \end{cases} \quad (12)$$

It is possible to capture the noncompensatory behavior of Scenario (2) in a modified, and still continuous, version of Scenario (1). Thus, to further explain the advantages of discontinuous representation of noncompensatory behavior in engineering design, we offer Scenario (3), where annual owning and operating costs influence utility in a compensatory fashion. Let $\eta \geq 0$, and define

$$u_\eta^S(a, g, p) = u(a, g, p) - \log(1 + e^{\eta s(a, p)}) \\ \vartheta_\eta^S = \eta - \log(1 + e^\eta) \quad (13)$$

where ϑ_η^S denotes the utility of the “outside good” or no-purchase option. This specification generates a compensatory logit model with choice probabilities

$$P_\eta^S(a, g, p) = \frac{e^{u_\eta^S(a, g, p)}}{e^{\vartheta_\eta^S} + e^{u_\eta^S(a, g, p)}} \quad (14)$$

This “smoothed” compensatory model is equivalent to Scenario (1)'s logit model (Eq. 9) when $\eta = 0$ and approaches the consider-then-choose probabilities (Eq. 12) as $\eta \uparrow \infty$.

3.3 Optimal Design-and-Pricing. Solving the combined choice and design model for the optimal vehicle with respect to price and acceleration requires a different approach for each of the above scenarios. Scenario (1) is solved as the smooth NLP

$$\max P(a, G(a), p)(p - c(a)) \\ \text{w.r.t. } 2.5 \leq a \leq 15, \quad p \geq 0 \quad (15)$$

using any available NLP solver, preferably one like SQP or IP methods that take advantage of derivative information.

Scenario (2) requires solving an optimization problem with a discontinuous objective that is not immediately solvable with smooth NLP techniques:

$$\max P^C(a, G(a), p)(p - c(a)) \\ \text{w.r.t. } 2.5 \leq a \leq 15, \quad p \geq 0 \quad (16)$$

A simple transformation casts Eq. (16), formally a nondifferentiable problem, as a standard smooth NLP. Because profits are zero for any design and price violating the budget rule ($k(G(a), p) > B$) but are positive for any design and price satisfying the budget rule ($k(G(a), p) \leq B$) such that $p > c(a)$, the budget rule *must* be satisfied by the optimal design and price. Thus Eq. (16) can be solved by solving the smooth NLP

$$\max P(a, G(a), p)(p - c(a)) \\ \text{w.r.t. } 2.5 \leq a \leq 15, \quad p \geq 0 \\ \text{s.to } B - k(G(a), p) \geq 0 \quad (17)$$

As described in Ref. [54], this technique does not easily generalize to other problems.

The contours of the objectives, the budget constraint, and the optimal solutions for Scenarios (1) and (2) are illustrated in Fig. 1. Solution details are given in Table 1. Equation (15) is solved by designing a vehicle with 0–60 acceleration time of 4.5 s (with a corresponding fuel economy of 10.2 mpg) and pricing this vehicle at roughly \$55,100. The expected profits with this design and price, normalized by market size, are roughly \$27,700. However, this vehicle does *not* satisfy the budget constraint. Thus, under consider-then-choose behavior, profits for this vehicle would be \$0. Acknowledging consider-then-choose behavior by solving

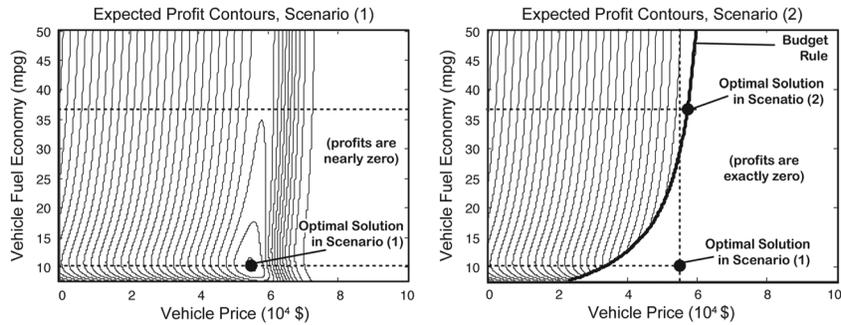


Fig. 1 Contours of profits optimal solutions to Eqs. (15) and (16). Green denotes profits that are nearly, or exactly, zero.

Table 1 Optimal solutions to Eqs. (15) and (16). All solutions computed with SNOPT's SQP algorithm. See also Fig. 1.

Scenario	Problem	a (s)	l/g (mpg)	p (\$)	$\pi(a, p)$	$\pi^c(a, p)$
(1) Compensatory choice	Equation (15)	4.5	10.2	\$55,100	\$27,700	\$0
(2) Noncompensatory choice	Equation (16)	13.4	36.5	\$57,200	\$22,900	\$22,900

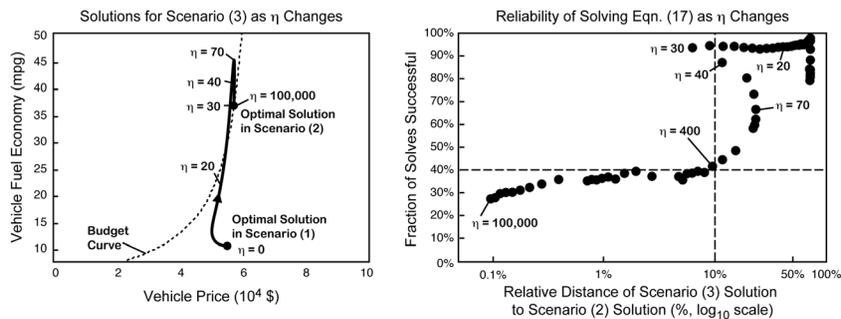


Fig. 2 Results for Scenario (3). *Left:* Optimal vehicle designs for Eq. (18) as η varies. Arrow indicates direction of increasing η . *Right:* Fraction of successful solves of Eq. (18) versus the distance of the optimal solution to the solution from Scenario (2).

Eq. (17) leads to a slightly higher price of \$57,200, but a radically different design: acceleration should be 13.4 s (with a fuel economy of 36.5 mpg). Optimal profits for this vehicle are roughly \$22,900. Though these profits are 17.3% lower than those expected under Scenario (1), if customers do, in reality, follow these screening rules, designing the vehicle under Scenario (1) will give profits of 0, in fact negative profits if R&D and manufacturing costs were included. Thus, significantly suboptimal designs may be obtained if consideration behavior exists and is ignored during product design.

Now, we consider Scenario (3), in which the budgeting is recognized to impact choice but is modeled in a compensatory, or continuous, fashion. For any given $\eta \geq 0$, the “smoothed” compensatory model presented in Eqs. (13) and (14) above results in the following optimal design problem:

$$\begin{aligned} \max \quad & P_{\eta}^S(a, G(a), p)(p - c(a)) \\ \text{w.r.t.} \quad & 2.5 \leq a \leq 15, \quad p \geq 0 \end{aligned} \quad (18)$$

As η ranges from 0 toward ∞ , this problem is a better and better approximation to the noncompensatory problem in Scenario (2). Specifically, Fig. 2 (left) illustrates that as η increases, the solutions to Eq. (18) approach the solution to Scenario (2). Thus, it is possible to represent the outcome of noncompensatory decisions with a compensatory model. However precisely rendering the

cut-off in preferences associated with a noncompensatory model is also associated with a significant decrease in computational performance. Figure 2 (right) shows that as the solutions to Scenarios (2) and (3) converge, SNOPT is increasingly unable to reliably solve Eq. (18). In particular, to obtain a Scenario (3) solution with a relative distance to Scenario (2)'s solution less than 10%, SNOPT succeeds in less than 40% of runs started from random initial conditions. With more complex and heterogeneous screening rules, these computational difficulties are intractable.

4 General Computational Methods

This section examines two different approaches for solving generic instances of Eq. (5). The first approach is to apply a GA directly to Eq. (5). We examine two techniques for constraint handling in GAs. The second approach is to apply an existing NLP method—Mathematical Programming with Complementarity Constraints (MPCC). These two approaches resulted in the testing of three methods, which we abbreviate P-GA (penalized GA), C-GA (constrained GA), and P-MPCC (relaxation with objective penalization). The two different approaches use different representations of the choice probabilities. GAs work directly with the discontinuous choice probabilities, illustrated in Fig. 3, left. Relaxation with complementarity represents the discontinuity by interpolating across additional dimensions, as illustrated in Fig. 3, right.

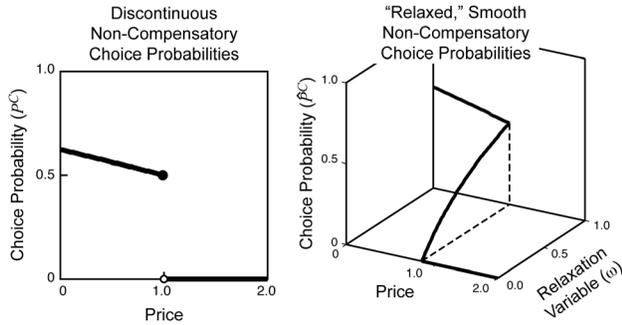


Fig. 3 Non-compensatory (left) and relaxed (right) choice probabilities for Eq. (4) based on Example 1, Sec. 2.

A smoothing method similar to Scenario (3) in Sec. 3 was also derived, implemented, and tested on the example in Sec. 5; see Ref. [55] for general information about smoothing. Smoothing can solve consider-then-choose design problems when implemented correctly, but with generally worse performance than the MPCC-relaxation method described below. Hence, we exclude this method for brevity; further information can be obtained from the authors.

4.1 Genetic Algorithms. GAs improve the objective value using only function evaluations, and thus are a natural candidate for problems with discontinuous objectives [26]. Solving optimal design problems with GAs requires handling design constraints in the GA [56]. We examine two methods for constraint handling: constraint penalization and feasibility projection.

Constraint penalization (P-GA) solves an unconstrained problem with some metric of constraint violation added into the objective and/or reproductive selection phase of the GA [57]. The selection and the scaling of the penalty parameter effects the balance between optimizing the objective and achieving feasibility [58], which is especially important when equality constraints are involved. We implement the penalty-based method proposed in Ref. [59]: for J products, the algorithm randomly draws a population of N design points, $(\mathbf{X}_n, \mathbf{p}_n) = \{(\mathbf{x}_{j,n}, p_{j,n})\}_{j=1}^J$. It then evaluates the profits for each member, $\pi_n = \hat{\pi}^C(\mathbf{X}_n, \mathbf{p}_n)$. Fitness values used in the GA are equal to profits if equality constraint violations are within a specified tolerance ϵ and inequality constraints are satisfied. Otherwise, fitness values for each member are equal to the negative of the relaxed constraint violation. The relaxation parameter ϵ is not updated during GA evolution, thus there is a limited ability to achieve strict feasibility.

The top 10% of population members, ranked according to fitness values, are copied into the next population (elitism) [26]. The remainder of the new population is determined by reproduction with crossover choosing parents via tournament selection [26]. The following criteria are used to decide which of a randomly drawn pair of members “wins” the tournament to be a parent: (i) if only one of the potential parents is feasible relative to the relaxed equality constraints, choose that member as the parent; (ii) if both members are feasible, select the one with higher profit; (iii) if the two members are both infeasible, select the one with lower constraint violation. As the model has continuous variables (e.g., fuel economy, acceleration, price), crossover is accomplished by taking a random convex combination of each variable value from the two parents. Mutations—random changes in some variable’s value—initially occur with a relatively large probability (0.5) and decrease in probability as the evolution proceeds. We empirically adjust the frequency of decreasing the mutation probability according to the number of constraints involved in the problem.

In the projection-based GA method (C-GA), we iterate with populations that have no infeasible members. A “constrained” or

projected GA first randomly generates an initial population, and projects it onto the feasible set. Solving Eq. (19)

$$\begin{aligned} \min \quad & \frac{1}{2} \|\mathbf{X}'_n - \mathbf{X}_n\|_2^2 \\ \text{w.r.t.} \quad & \mathbf{l}_j \leq \mathbf{x}'_{j,n} \leq \mathbf{u}_j \\ \text{s.to} \quad & \mathbf{c}_j^E(\mathbf{x}'_{j,n}) = \mathbf{0}, \mathbf{c}_j^I(\mathbf{x}'_{j,n}) \geq \mathbf{0}, \text{ for all } j \end{aligned} \quad (19)$$

finds the closest feasible designs \mathbf{X}'_n to \mathbf{X}_n . We can then replace \mathbf{X}_n with \mathbf{X}'_n in the population, keeping \mathbf{p}_n the same. The algorithm then computes profitability $\pi_n = \pi^C(\mathbf{X}_n, \mathbf{p}_n)$ for all members, having already assured their feasibility, and uses profits as fitness values. As with the penalized GA, elitism and reproduction through tournament selection with crossover and mutation are used to define new populations that are also projected onto the feasible set.

Termination Criteria. In both GA implementations, we terminate when the average fitnesses over the whole population in two successive iterations are sufficiently close (e.g., absolute difference smaller than 10^{-5}).

4.2 MPCC Relaxation. This section describes an NLP formulation of Eq. (5) based on the following steps:

- (1) *Relax* the discontinuous choice probabilities by introducing slack variables that can represent satisfaction or violation of each screening rule for every individual and product;
- (2) *Relate* the relaxed choice probabilities to the original consider-then-choose choice probabilities by adding *complementarity constraints* representing *choice consistency*;
- (3) *Optimize* designs, prices, and the slack variables subject to these constraints with suitable methods for smooth NLPs.

Specifically, an MPCC can be derived by relaxing the choice probabilities using a vector of relaxation variables $\boldsymbol{\omega} = \{\omega_{i,j,r} : 1 \leq i \leq I, 1 \leq j \leq J, 1 \leq r \leq R_j\}$, where $\omega_{i,j,r} \in [0, 1]$. Let

$$\hat{P}_{ij}^C(\boldsymbol{\omega}, \mathbf{X}, \mathbf{p}) = \frac{\left(\prod_{r=1}^{R_i} \omega_{i,j,r} \right) e^{u_i(\mathbf{x}_j, p_j)}}{1 + \sum_{k=1}^J \left(\prod_{r=1}^{R_i} \omega_{i,k,r} \right) e^{u_i(\mathbf{x}_k, p_k)}} \quad (20)$$

and associate $\omega_{i,j,r} = 1$ with product j satisfying individual i 's r th screening rule, and $\omega_{i,j,r} = 0$ with product j violating this screening rule. The products $\prod_{r=1}^{R_i} \omega_{i,j,r}$ define consideration: $\prod_{r=1}^{R_i} \omega_{i,j,r} = 1$ means individual i considers product j , and $\prod_{r=1}^{R_i} \omega_{i,j,r} = 0$ means individual i does not consider product j . $\boldsymbol{\omega}$ is called (*strictly*) *choice-consistent with* (\mathbf{X}, \mathbf{p}) if $\omega_{i,j,r} = 1$ for all (i,j,r) such that $s_{i,r}(\mathbf{x}_j, p_j) \leq 0$ and $\omega_{i,j,r} = 0$ for all (i,j,r) such that $s_{i,r}(\mathbf{x}_j, p_j) > 0$. The choice probabilities in Eq. (20) are exactly equal to the consider-then-choose choice probabilities Eq. (3) when $\boldsymbol{\omega}$ is strictly choice-consistent. $\boldsymbol{\omega}$ is called *relaxed choice-consistent with* (\mathbf{X}, \mathbf{p}) if $\omega_{i,j,r} = 1$ for all (i,j,r) such that $s_{i,r}(\mathbf{x}_j, p_j) < 0$ and $\omega_{i,j,r} = 0$ for all (i,j,r) such that $s_{i,r}(\mathbf{x}_j, p_j) > 0$. Vectors $\boldsymbol{\omega}$ that are relaxed choice-consistent solve the “mixed complementarity problem” (MCP)

$$0 \leq \omega \leq 1 \perp s \quad \equiv \quad \begin{cases} \omega = 1 & \text{if } s < 0 \\ \omega \in [0, 1] & \text{if } s = 0 \\ \omega = 0 & \text{if } s > 0 \end{cases} \quad (21)$$

MCP's are a generalization of the KKT conditions for constrained optimization; see Refs. [60–64] for more information on MCPs. Relaxed choice-consistency and the associated MCP enable an NLP approach to solving Eq. (5).

An optimal design problem that uses Eq. (20) in place of Eq. (3), adds the relaxation variables $\boldsymbol{\omega}$ as problem variables, and

constrains them to satisfy the MCPs $0 \leq \omega_{i,j,r} \leq 1 \perp s_{i,r}(\mathbf{x}_j, p_j)$ is an MPCC [65] with relaxed choice-consistent solutions (see Eq. (34) and Lemma 2 in Appendix A). This is a relaxation of Eq. (5): the optimum is always at least as great as that of Eq. (5) and any (local) solution that is strictly choice-consistent is also a (local) solution to Eq. (5) (Lemma 3, Appendix A).

The difficulty is properly addressing the complementarity constraints from Eq. (21). While MPCCs have previously been considered to be difficult-to-solve, recent research has identified appropriate ways in which standard NLP methods like IP and SQP can be used to solve MPCCs; see Refs. [66–68] or the review in Ref. [65]. Two major approaches to solving MPCCs with complementary constraints have emerged: objective penalization [66,67] and adding constraints that reflect complementarity [68]. Penalization has been successfully applied to chemical process design with nonsmooth dynamics [27–29], among other areas. After implementing both approaches, we observed better performance with penalization. We use the penalized MPCC (P-MPCC) formulation

$$\max \begin{cases} \hat{\pi}^C(\mathbf{w}, \mathbf{X}, \mathbf{p}) + \tau \sum_{i,j,r} w_{i,j,r} \\ -M \sum_{i,j,r} (w_{i,j,r} s_{i,j,r}^+ + v_{i,j,r} s_{i,j,r}^-) \end{cases} \quad (22)$$

$$\text{w.r.t. } \mathbf{1}_j \leq \mathbf{x}_j \leq \mathbf{u}_j, p_j \geq 0 \text{ for all } j \quad (23)$$

$$w_{i,j,r}, v_{i,j,r}, s_{i,j,r}^+, s_{i,j,r}^- \geq 0 \text{ for all } i, j, r \quad (24)$$

$$\text{s.to } \mathbf{c}_j^E(\mathbf{x}_j) = \mathbf{0}, \mathbf{c}_j^I(\mathbf{x}_j) \leq \mathbf{0} \text{ for all } j \quad (25)$$

$$w_{i,j,r} + v_{i,j,r} = 1 \text{ for all } i, j, r \quad (26)$$

$$s_{i,j,r}(\mathbf{x}_j, p_j) - s_{i,j,r}^+ + s_{i,j,r}^- = 0 \text{ for all } i, j, r \quad (27)$$

$$\sum_{i=1}^I \left(\prod_{r=1}^{R_i} w_{i,j,r} \right) \geq 1 \text{ for all } j \quad (28)$$

for some $\tau, M > 0$, explained in detail below. In our explanation we neglect the indices i, j, r for clarity; it should be understood that our description applies for all i, j, r .

Four sets of slack variables represent the MCP Eq. (21): w , v , s^+ and s^- (all non-negative). The variable w is the relaxation variable ω . Equation (26) ensures that $0 \leq v = 1 - w$, and thus $0 \leq w \leq 1$. Equation (27) ensures that, when feasible and complementary, s^+ and s^- are the positive and negative parts of $s(\mathbf{x}, p)$, defined as

$$|s(\mathbf{x}, p)|_+ = \max\{0, s(\mathbf{x}, p)\} \quad (29)$$

$$|s(\mathbf{x}, p)|_- = -\min\{0, s(\mathbf{x}, p)\} \quad (30)$$

The conditions $ws^+ = 0$ and $vs^- = 0$ then imply w solves the MCP Eq. (21). These conditions are enforced in Eqs. (22)–(28) through subtracting $M \sum (ws^+ + vs^-)$ from the objective. For large enough M , this penalization forces both ws^+ and vs^- to be zero, thus obtaining both complementarity and the original unpenalized objective value [67]. This penalization also enforces complementarity between s^+ and s^- at any local solution. The term $\tau \sum w$ in the objective enforces convergence to *strictly* choice-consistent solutions. Lemma 4, Appendix A, proves that there is a value of τ large enough so that any solution to Eqs. (22)–(28) is strictly choice-consistent. Finally, Eq. (28) ensures that every product is considered by some individual; this rules out “trivial” solutions in which products are not considered at all. Including this constraint improves convergence to good solutions of Eqs. (22)–(28).

5 Example: Vehicle Portfolio Design With Heterogeneous Screening Rules

This section presents an extension of the vehicle design example presented in Sec. 3. Section 5.1 explains the formulation of the heterogeneous choice model and multiple product offerings used in this extension. This extension presents the challenges of optimization with multiple local solutions. We use this example to compare performance of P-GA, C-GA, and P-MPCC as discussed in Sec. 4. We discuss the characteristics of the methods in terms of both the quality of the solutions (profitability) and the cost (time and function evaluations) in Sec. 5.3.

5.1 Heterogeneous Choice Model. There are I individuals, and each individual $i \in \{1, \dots, I\}$ has an individual-specific screening rule

$$s_i(a, g, p) = R_i p + m_i p_i^G g - B_i \quad (31)$$

for interest rate r_i , annual miles travelled m_i , gasoline price p_i^G and annual budget level B_i . Each individual also has different compensatory preferences over fuel consumption, acceleration, and price, embodied by the utility function

$$u_i(a, g, p) = \beta_{p,i} p + \beta_{g,i} g + \frac{\beta_{a,i}}{a} + \vartheta_i \quad (32)$$

The coefficients in this equation are as follows [49]: $\beta_{p,i} = -|4.6 + 0.2/\iota_i - 0.4N_{p,i}|$ where ι_i denotes household income; $\beta_{g,i} = -36.8 + 2.2N_{g,i}$; $\beta_{a,i} = 11.3 + 0.3N_{a,i}$; and $\vartheta_i = 23.2 + 0.5N_{0,i}$. The variables $N_{p,i}$, $N_{g,i}$, $N_{a,i}$, and $N_{0,i}$ each denote independent samples from a standard normal distribution.

The individualized budget screening rule parameters are sampled as follows: We sample interest rates r_i from a uniform distribution on $[0.03, 0.08]$ (3%–8% per year). We restrict income to be larger than \$50,000, assuming that new car purchases are dominated by households having annual incomes larger than this value, and sample ι_i from an empirical frequency distribution based on data in the 2006 Current Population Survey [69]. This income is applied in both the utility model and estimates of miles driven and annual budget. Annual miles driven are assumed to follow a power law with income: $m_i = 6639 \iota_i^{0.288} e^{0.253N_{m,i}}$ where $N_{m,i}$ is a sample from a standard normal distribution. The coefficients were estimated based on the data in the National Household Transportation Survey [70]. The annual budget is assumed to follow a linear relationship with income: $B_i = 0.5 + 0.06\iota_i + 0.0512N_{B,i}$ where again $N_{B,i}$ is a sample from a standard normal distribution. The coefficients in this model were estimated based on data in the Consumer Expenditure Survey [71]. The loan period (N) and the price of gasoline (p^G) are assumed to be fixed at 10 and \$3.5, respectively, for all individuals.

The optimal vehicle portfolio design problem with this heterogeneous population is then

$$\begin{aligned} \max \quad & \sum_{j=1}^J \left(\sum_{i=1}^I P_{i,j}^C(\mathbf{a}, \mathbf{g}, \mathbf{p}) \right) (p_j - c(a_j, g_j)) \\ \text{w.r.t.} \quad & 2.5 \leq a_j \leq 15, g_j \geq 0, p_j \geq 0 \text{ for all } j \\ \text{s.to} \quad & p_j - c(a_j, g_j) \geq 0 \text{ for all } j \\ & g_j - G(a_j) = 0 \text{ for all } j \end{aligned} \quad (33)$$

Notice that we include fuel consumption as a variable, and constrain it to satisfy Eq. (7). This defines an optimal design problem with constraints.

5.2 Computational Details. Instances of Eq. (33) with $J = 1, 2, 5, 10$ vehicles and $I = 1, 5, 10, 20, 30, 40, 50$ individuals were tested, enabling an observation of performance on different

problem scales. Implementations of P-GA, C-GA, and P-MPCC for Eq. (33) were written in C code. All GA runs had an initial mutation probability of 0.5 and population size of 50. The NLPs in C-GA and P-MPCC were solved using the SQP solver SNOPT [23] with relative optimality and feasibility tolerances set to 10^{-6} . SNOPT is a widely tested state-of-the-art large-scale SQP solver that performs well for constrained design problems. Because it has an elastic mode technique to handle infeasible quadratic subproblems, it is particularly well suited to solving MPCCs with either objective penalization or bilinear complementarity constraints [68]. See Ref. [22] for a detailed treatment of other NLP solvers. KNITRO [24] and IPOPT [72] can also solve MPCCs, but our testing found SNOPT to be faster and more reliable. We did not test Augmented Lagrangian methods [25], but recent research suggests they could perform well on MPCCs [73].

All P-MPCC runs had the τ parameter set at ten to ensure strict choice-consistency and the M parameter set at 1000 to ensure complementarity. For P-GA and P-MPCC, 1000 trials were undertaken by drawing random initial conditions. C-GA uses only 100 random trials; this method was more time consuming and this number of runs was sufficient to represent performance. All computations were undertaken on a single Mac Pro tower with 2, quad-core 2.26 GHz processors and 32 GB of RAM running OS X (10.6.8). We present illustrative results below; complete statistics for all cases are available by request.

5.3 Results of Method Comparison. Figure 4 illustrates the best solutions found by each method on two cases: 5 products with 10 individuals, $(J,I)=(5,10)$ on the left, and 10 products with 50 individuals, $(10,50)$, on the right. Figure 4, left, also illustrates the corresponding budget curves for the 10 individuals with gray lines; these are left out of the right figure for clarity. The multiple points plotted for each method correspond to the design and price of each vehicle offered in the portfolio. Only one of the two design variables—fuel economy—is plotted because acceleration is a function of fuel economy in this model. Both figures also list the corresponding profits per vehicle, in parentheses on the figure legends.

Figure 4 shows that multiple vehicles are offered in the best solution found by any method in both cases. The budget curves are “active” at the best solutions found, as illustrated left in the figure. P-MPCC leads to the smallest number of vehicles designed: with $(J,I)=(5,10)$, P-MPCC finds a best solution with only three distinct vehicles; with $(J,I)=(10,50)$ has only two. P-GA and C-GA lead to designs of four and five vehicles in the $(5,10)$ case and ten and nine in the $(10,50)$ case, respectively. While all solutions recommend designing vehicles with different fuel economies, C-GA and P-MPCC capture the breadth of the optimal portfolio design in both cases finding a best solution that offers vehicles with both

low and high fuel economies. In both cases, P-GA leads to the design of vehicles with similar fuel economies and prices. The solutions have high prices—between \$40k and \$50k—relative to recent real vehicle prices. Section 6 discusses why this occurs.

Two metrics, feasibility and optimality (profitability), serve to quantify solution quality over all solutions found with each method. **Feasibility:** P-MPCC and C-GA find solutions that are more feasible than P-GA. In P-MPCC and C-GA, SNOPT ensures the relative violation of the constraint on fuel consumption, $\max_j\{|g_j - G(a_j)|/G(a_j)\}$, to less than 10^{-4} . This “relative constraint violation” is different from the “relative feasibility tolerance” used by SNOPT that ensures [23]

$$\max_j\{|g_j - G(a_j)|\} / \max_j\{\max\{|a_j|, |g_j|, |p_j|\}\}$$

is less than 10^{-6} . Constraint penalization in P-GA is able to obtain a relative constraint violation under 3×10^{-3} at solutions but was not able to achieve tighter tolerances on feasibility without significantly sacrificing objective value (profits).

Optimality: Figure 5 compares profitability for two representative cases, $(J,I)=(5,10)$ and $(J,I)=(10,50)$, across all three methods and trials. The X-axis presents the relative difference between profitability of trial solutions and π_*^C , the solution with the highest profits found for a particular scenario with a relative constraint violation less than 10^{-4} . The lines plot the cumulative distribution functions (CDFs) of profitability, normalized by π_*^C . The higher and steeper the plotline, the better-performing the method. The intersection with the vertical axis marks the percentage of trials that computed a value of profits within 1% of π_*^C .

C-GA: In both cases, C-GA only finds solutions with an objective level within 95% of π_*^C , and solutions within 1% π_*^C in roughly 70% of trials. Feasibility (with relative constraint violation $<10^{-4}$) is guaranteed by the feasibility projection method. Thus, C-GA reliably finds feasible solutions very close to π_*^C .

P-GA: Almost all solutions found with P-GA achieve profits higher than 80% of π_*^C , but P-GA does not find a solution within 1% of π_*^C in either case. The ability to achieve a better profit level (e.g. 90% of π_*^C decreases as problem size increases. For example, in the $(5,10)$ case, over 90% of the solutions have a profit level of 90% of π_*^C , while in the $(10,50)$ case, only 30% of solutions do. Constraint violations often several orders of magnitude larger than solutions found with C-GA or P-MPCC.

P-MPCC: P-MPCC has good performance in the $(5,10)$ case, 10% of trials result in solutions within 1% of π_*^C , and 90% of trials result in solutions within 90% of π_*^C . In the $(10,50)$ case, P-MPCC is strongly attracted to a local optima with

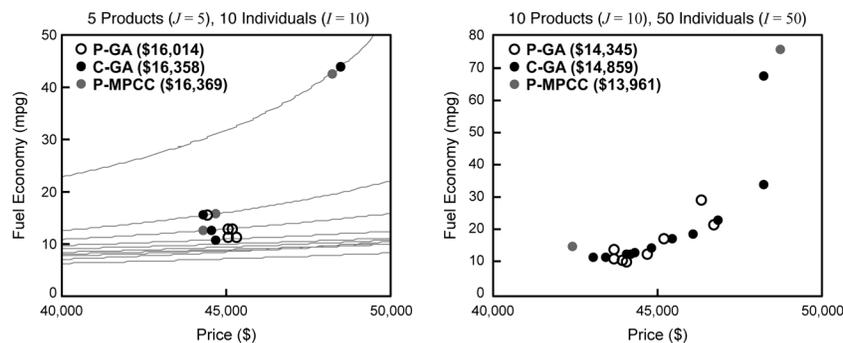


Fig. 4 Vehicle design solutions for the three methods across two cases; note the different scales on the vertical axis in left and right figures. Gray lines on the left figure denote the budget curves $R_i p + m_i p^G g = B_i$. Best vehicle portfolio profits, per vehicle sold, found by each method are in parentheses.

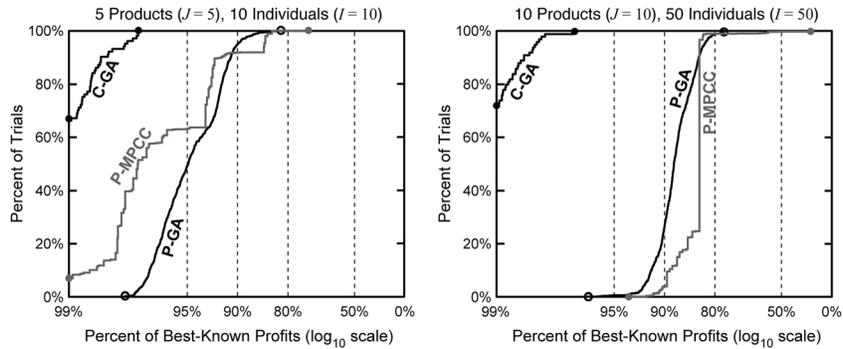


Fig. 5 Empirical CDF of solution profits found relative to the best-known profit in two cases: $(J,I) = (5,10)$ (left) and $(J,I) = (10,50)$ (right).

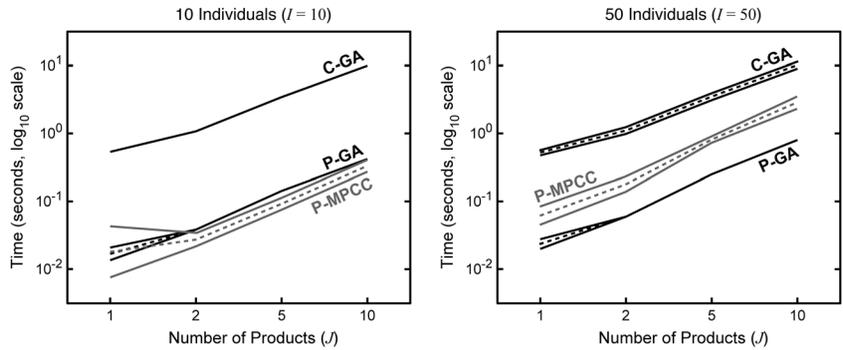


Fig. 6 Solution times for Eq. (33) for two population sizes. Means plotted with dashed lines, with solid lines illustrating one standard deviation above and below the mean.

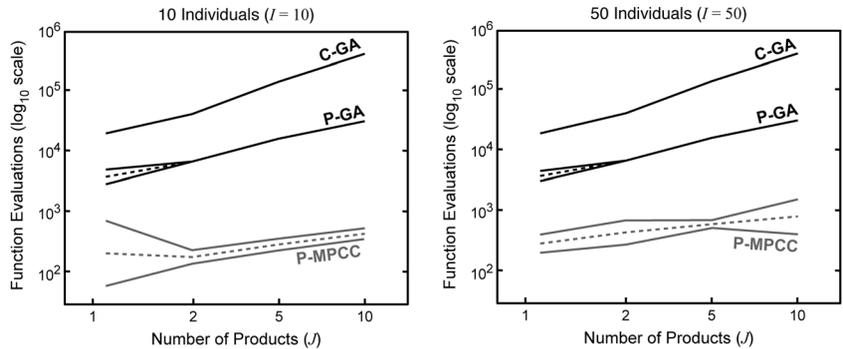


Fig. 7 Function evaluation counts for Eq. (33) for two population sizes. Means plotted with dashed lines, with solid lines illustrating one standard deviation above and below the mean.

profits at 83% of π_*^C . Note that P-MPCC computed profits within 80% of the best-known profits in almost all trials in the larger problem but does not find a solution within 95% of π_*^C .

Two metrics serve to quantify computational cost: compute time and number of function evaluations. **Compute time:** Figure 6 plots compute time (in seconds) for all three methods with $I=10$ and $I=50$ and various numbers of products. The mean compute time is plotted with a dashed line, and solid lines illustrate one standard deviation above and below the mean. C-GA has the most expensive computations, requiring 1-10 s for both values of I . For the $I=10$ case, the compute time required by P-GA and P-MPCC

is similar, with P-MPCC faster (within one standard deviation) for more than one product. P-GA is faster than P-MPCC for cases with $I=50$. Compute time for both P-GA and C-GA changed significantly with J , but not with I (compare Fig. 6 left and right). Compute times for P-MPCC changed significantly with both I and J .

Function evaluations: Figure 7 illustrates mean and deviation in number of function evaluations for all methods. P-MPCC required the fewest function evaluations, roughly 2 orders of magnitude fewer than P-GA and three less than C-GA. C-GA required the most function evaluations, 10,000 to 100,000, for both $I=10$ and $I=50$. P-GA required fewer function evaluations than C-GA, but still more than P-MPCC. Function evaluations for both P-GA

and C-GA changed significantly with J , but not with I (compare Fig. 7 left and right). Function evaluations for P-MPCC again changed with both I and J .

6 Discussion of Results

The best solutions for the three methods illustrated in Fig. 4 indicate that noncompensatory screening rules can play an important role in design optimization, as every best solution involved vehicles designed at the boundary of consideration, effectively making screening rules “active” in the solutions. This reinforces the conclusion from Sec. 3 that neglecting to model noncompensatory screening rules can result in missed profits and misleading forecasts of design success. Adding heterogeneous screening rules to an already complex design optimization presents new challenges. Each method for solving the problem offers a different “best” set of vehicles, with some overlap. If an analyst presented these results to management at an automotive firm, it would be difficult to explain which set of results is the right one to use. Highest profitability is one metric, but it is also important that vehicles be distinct from one another. Figure 4 shows that, in both cases investigated, P-GA tends to cluster vehicle designs (acceleration and fuel economy) and prices in the vehicle portfolio too close together to actually be useful to designers. This behavior may be improved with the application of additional constraints on portfolio design and/or a design model with more variables. P-MPCC locates solutions with close-to-optimal profits with fewer vehicles that are significantly different from each other. If this trend extrapolates beyond this example, it may be advantageous to use the P-MPCC results for portfolio reasons. Needing to carry fewer vehicle models for the same profits can be more cost-competitive. The other methods, P-GA and C-GA, can be used to confirm the results of P-MPCC with further constraints on the number of vehicles produced and/or the required “distance” between fuel economy in the resulting vehicles designed.

The results do not point to an absolute conclusion regarding the best method for solving an optimal design problem with a consider-then-choose demand model. P-MPCC can quickly compute good solutions to Eq. (33) when the heterogeneity in screening is small (for the example in Sec. 5, this is represented by small I). C-GA has the highest likelihood of finding good, feasible solutions across problem sizes, but takes significantly longer than other methods. P-GA solves the example problem fastest overall, but provides worse solutions than P-MPCC on small problems and only slightly better solutions on larger problems. P-GA has higher constraint violations at least one order of magnitude larger than those of C-GA and P-MPCC. In our example, the constraint violations are probably not significant. However, if feasibility was more important to the application, these higher constraint violations may rule out P-GA as a viable method. We also suspect that the constraint penalization in P-GA is interfering with convergence to global solutions; see Fig. 5. If so, attaining “reasonable” levels of feasibility is not the only concern with P-GA.

Without a complete theoretical analysis, we do not know with certainty how the methods examined will perform on other problems. Most observations are consistent with the general knowledge about the performance of evolutionary and derivative-based NLP algorithms. Derivative-based NLP algorithms can quickly find local solutions with few function evaluations, but may have difficulty finding global optima. They may not scale well with the number of variables and constraints without additional structure (e.g., linearity). GAs are robust to problem irregularity but perform best with cheap function evaluations and may have difficulty achieving strict feasibility when it is hard to explicitly restrict populations to the feasible set. These algorithmic features affect our results, suggesting how computational reliability and burden could be different for different design problems. For example, P-GA may perform better for problems with inactive inequality constraints, as the constraint penalization may interfere less with progress towards global optima. If the constraints are linear, even

if equalities, then projection onto the feasible set is nearly trivial and C-GA may be much faster than demonstrated here. Should function evaluations be significantly more expensive, perhaps involving simulation of choice probabilities, design constraints, or performance maps, P-MPCC may be more competitive with C-GA and P-GA. Specifically, Fig. 7 shows that P-GA and C-GA require up to three orders of magnitude more function evaluations than P-MPCC. In our example, this was not a significant expense because the function evaluations required to solve Eq. (33) are relatively simple.

A hybrid approach might take advantage of the best features of P-GA and P-MPCC. For example, P-GA could generate reasonable candidate solutions that are “refined” by P-MPCC every ten generations. With “good” starting conditions provided by P-GA, as well as strict limits on number of iterations taken in P-MPCC, a hybrid method might be able to rapidly identify global optima with a precision that is expensive to obtain with P-GA.

Finally, we note that the particular example in Secs. 3 and 5 has optimal prices significantly higher than those observed in real market data. These higher prices are due to our assumption of a ten-year loan period that makes expensive vehicles more affordable. Shorter loan periods would decrease individual’s purchasing power, effectively shifting the curves illustrated in Fig. 4, left, to the left along the X-axis. This would thus decrease optimal vehicle prices, making the results more consistent with real vehicle prices. Section 5 also restricts attention to a higher income bracket for new vehicle purchasers than real markets, which would tend to increase optimal vehicle prices.

7 Conclusions

This article has introduced consider-then-choose models to engineering design optimization. Consider-then-choose models are a two-stage variant of the generalized linear models commonly used in discrete choice analysis (DCA) and represent a noncompensatory decision process commonly observed in complex product purchases. A simple example illustrated that consideration can impact design, justifying the potential benefits of using this type of model in engineering design. To support further research in this area, we investigated computational methods suitable for solving optimal design problems with consider-then-choose models where consideration is defined by conjunctive screening rules. Three methods are discussed: two genetic algorithms (GAs) using different constraint handling strategies (C-GA, P-GA) and a nonlinear programming (NLP) method based on a novel relaxation using complementarity constraints (P-MPCC). These methods are tested and compared using a multiple product, heterogeneous population vehicle design example. The comparison on our example suggests each approach has benefits, with GAs being a superior method for larger scale instances but P-MPCC performing best on smaller scale instances.

This work has a number of limitations and areas for future work. One limitation of the work is that it includes submodels that are illustrative, for example, the preference model. Therefore, it is not possible to draw actual recommendations or conclusions regarding vehicle design from this study. This is an area of expansion for possible future work. Another area for future work is to understand why the relaxation approach finds optimization solutions with smaller product portfolios than the GA approaches. We speculate this evolves from a combination of the random initial population used in the GAs and the potential tendency of NLPs to converge to solutions with large basins of attraction. Once the mechanism for this behavior is understood, it may be possible to harness it to optimize product portfolios that are purposefully larger or smaller. The P-MPCC and CGA performance statistics are, of course, tied to the SNOPT solver that was used to implement them. Solvers other than SNOPT were investigated but found to be slower and less reliable as discussed briefly in Sec. 5.2. However, further approaches could be investigated. A limitation also exists in comparing the methods on a problem

where the global optimum is unknown. Although we can compare the results and performance of the algorithms, it is impossible to know that the best optimum determined in our tests is truly the global optimum. To test this fully would require using a problem where the global optimum is somehow predetermined. Lastly, the P-MPCC approach, which uses an NLP formulation, cannot handle discrete variables in its current form, unlike the GAs. Modifications such as Branch-and-Bound [74] could be implemented to extend the capabilities of the P-MPCC method.

Despite the computational challenges of including heterogeneous consideration rules in design optimization, this paper demonstrates that consideration can be modeled, and potentially produces beneficial insights into design problems. It also highlights the challenge of choosing one method as the “right” optimization method, when different approaches offer different results.

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Nomenclature

- c_j = unit cost of product j .
- c_j^E, c_j^I = equality and inequality constraint functions (respectively) for product j .
- C_i = individual i 's consideration set, defined in Eq. (1).
- I = number of individuals (indexed by i).
- \mathbf{l}, \mathbf{u}_j = upper and lower bounds on vector of attributes of product j .
- J = number of products (indexed by j).
- p_j = price of product j .
- \mathbf{p} = vector of prices for all products.
- $\hat{P}_{i,j}^C$ = relaxed consider-then-choose choice probability that individual i chooses product j , defined in Eq. (20).
- $P_{i,j}^C$ = consider-then-choose choice probability that individual i chooses product j , defined in Eq. (3). We also use P^C in single-product, single-individual cases.
- π^C = profits under a consider-then-choose model; defined in Eq. (6).
- s_i = individual i 's screening rules; see Eq. (1).
- u_i = individual i 's utility function for any considered product.
- ϑ_i = individual i 's utility of the “outside good” or “no purchase” utility.
- \mathbf{x}_j = vector of attributes of product j , not including price.
- \mathbf{X} = “matrix” of attribute vectors for all products.
- $\omega_{i,j,r}$ = “Slack” variables used in the relaxed consider-then-choose choice probabilities.

Additional Nomenclature used in the Examples in Secs. 3 and 5:

- a_j = vehicle j 's acceleration (without index for single-vehicle example).
- g_j = vehicle fuel consumption (without index for single-vehicle example).
- e_j = vehicle fuel economy (without index for single-vehicle example); equals $1/g_j$.
- P = smooth logit choice probabilities.
- t_i = individual i 's annual income.
- B_i = individual i 's budget level.
- r_i = individual i 's annual percentage interest rate on vehicle financing
- N_i = individual i 's financing period
- m_i = individual i 's annual miles traveled
- p_i^G = individual i 's price of gasoline

- k = operating costs, defined in Eq. (11)
- R = financing amortization factor on MSRP (p), defined in Eq. (11)
- π = profits using a simple Logit model

Appendix A: Methodological Mathematical Results

LEMMA 1. (i) For any (\mathbf{X}, \mathbf{p}) , there is a unique vector $\bar{\omega}(\mathbf{X}, \mathbf{p}) \in \Omega$ that is strictly choice-consistent with (\mathbf{X}, \mathbf{p}) . (ii) If ω is strictly choice-consistent at (\mathbf{X}, \mathbf{p}) then $P_j^C(\mathbf{X}, \mathbf{p}) = \hat{P}_j^C(\omega, \mathbf{X}, \mathbf{p})$. (iii) Subsequently, $\pi^C(\mathbf{X}, \mathbf{p}) = \hat{\pi}^C(\bar{\omega}(\mathbf{X}, \mathbf{p}), \mathbf{X}, \mathbf{p})$.

Proof. We prove (i), with (ii) and (iii) easy corollaries. Suppose that there are two distinct vectors $\omega, \omega' \in \Omega$ that are both strictly choice-consistent with (\mathbf{X}, \mathbf{p}) . Then for some (i, j, r) , $\omega_{i,j,r} \neq \omega'_{i,j,r}$. But by strict choice consistency, $\omega_{i,j,r} = 1 = \omega'_{i,j,r}$ if $s_{i,r}(\mathbf{x}_j, p_j) \leq 0$ and $\omega_{i,j,r} = 0 = \omega'_{i,j,r}$ if $s_{i,r}(\mathbf{x}_j, p_j) > 0$. This contradiction proves that $\omega = \omega'$. \square

Consider the following version of Eqs. (22)–(28):

$$\begin{aligned} \max \quad & \hat{\pi}^C(\omega, \mathbf{X}, \mathbf{p}) \\ \text{w.r.t.} \quad & \omega_{i,j,r} \in [0, 1] \text{ for all } i, j, r \\ & \mathbf{l}_j \leq \mathbf{x}_j \leq \mathbf{u}_j, p_j \geq 0 \text{ for all } j \\ \text{s.to} \quad & \mathbf{c}_j^E(\mathbf{x}_j) = \mathbf{0}, \mathbf{c}_j^I(\mathbf{x}_j) \leq \mathbf{0} \text{ for all } j \\ & \sum_{i=1}^I \left(\prod_{r=1}^{R_i} \omega_{i,j,r} \right) \geq 1 \text{ for all } j \\ & 0 \leq \omega_{i,j,r} \leq 1 \perp s_{i,r}(\mathbf{x}_j, p_j) \text{ for all } i, j, r \end{aligned} \quad (34)$$

Equation (34) can be solved by the penalization method formulated in Eqs. (22)–(28) or with other techniques; e.g., see Ref. [54]. Because of this flexibility in solution technique, it is useful to state important technical results in terms of Eq. (34).

LEMMA 2. If $(\omega, \mathbf{X}, \mathbf{p})$ is feasible for Eq. (34), then ω is relaxed choice-consistent with (\mathbf{X}, \mathbf{p}) .

Proof. For feasible $(\omega, \mathbf{X}, \mathbf{p})$, this follows directly from the definition of relaxed choice-consistency and the MCP $0 \leq \omega_{i,j,r} \leq 1 \perp s_{i,r}(\mathbf{x}_j, p_j)$. \square

LEMMA 3. $\hat{\pi}^{CC,*} \geq \pi^{CC,*}$ where $\hat{\pi}^{CC,*}$ is the optimal value of Eq. (34) and $\pi^{CC,*}$ is the optimal value of Eq. (5). Moreover, if $(\omega, \mathbf{X}, \mathbf{p})$ is a local solution of Eq. (34) and ω is strictly choice-consistent with (\mathbf{X}, \mathbf{p}) , then (\mathbf{X}, \mathbf{p}) is a local solution to Eq. (5).

Proof. By Lemma 1(ii), $\pi^C(\mathbf{X}, \mathbf{p}) = \hat{\pi}^C(\bar{\omega}(\mathbf{X}, \mathbf{p}), \mathbf{X}, \mathbf{p})$. Thus, because $(\bar{\omega}(\mathbf{X}, \mathbf{p}), \mathbf{X}, \mathbf{p})$ is feasible for Eq. (34) for any feasible (\mathbf{X}, \mathbf{p}) , the optimal value $\pi^{CC,*}$ can be attained in Eq. (34) by some feasible $(\omega, \mathbf{X}, \mathbf{p})$. Thus, the optimal value of Eq. (34) must be at least $\pi^{CC,*}$.

For the second claim, assume that $(\omega, \mathbf{X}, \mathbf{p})$ is a local solution of Eq. (34) and ω is strictly choice-consistent with (\mathbf{X}, \mathbf{p}) . By Lemma 1, $\omega = \bar{\omega}(\mathbf{X}, \mathbf{p})$ and $\pi^C(\mathbf{X}, \mathbf{p}) = \hat{\pi}^R(\omega, \mathbf{X}, \mathbf{p})$. Local optimality means that $\hat{\pi}^R(\omega, \mathbf{X}, \mathbf{p}) \geq \hat{\pi}^R(\omega', \mathbf{X}', \mathbf{p}')$ for all $(\omega', \mathbf{X}', \mathbf{p}')$ in a (feasible) neighborhood \mathcal{U} of $(\omega, \mathbf{X}, \mathbf{p})$. Let \mathcal{D} be the projection of \mathcal{U} onto the \mathbf{X} and \mathbf{p} coordinates. Because $(\bar{\omega}(\mathbf{X}', \mathbf{p}'), \mathbf{X}', \mathbf{p}') \in \mathcal{U}$ for all $(\mathbf{X}', \mathbf{p}') \in \mathcal{D}$,

$$\begin{aligned} \pi^C(\mathbf{X}, \mathbf{p}) &= \hat{\pi}^R(\omega, \mathbf{X}, \mathbf{p}) \\ &\geq \hat{\pi}(\bar{\omega}(\mathbf{X}', \mathbf{p}'), \mathbf{X}', \mathbf{p}') = \pi^C(\mathbf{X}', \mathbf{p}'). \end{aligned}$$

Thus (\mathbf{X}, \mathbf{p}) is locally optimal for Eq. (5). \square

LEMMA 4. Add $\tau \sum_{i,j,r} \omega_{i,j,r}$ to the objective in Eq. (34) for $\tau \geq 0$. (i) Any strictly choice-consistent, strict local solution to Eq. (34) solves this penalized problem. (ii) Moreover, for large enough $\tau > 0$, local solutions to this penalized problem are strictly choice-consistent.

Proof. The forward implication (i) is trivial: any feasible change in the slack variables $\omega_{i,j,r}$ (if any) decreases them, and results in a strict local decrease in $\hat{\pi}^C$ and does not increase $\tau \sum_{i,j,r} \omega_{i,j,r}$. (ii) follows from the fact that, for large enough τ , $\hat{\pi}^C + \tau \sum_{i,j,r} \omega_{i,j,r}$ is increasing in $\omega_{i,j,r}$ for all (i,j,r) . \square

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